THE ANALYSIS OF THE INFLUENCE OF TEMPERATURE AND ATMOSPHERIC PRESSURE MEASUREMENT ERRORS ON THE FLIGHT ALTITUDE DETERMINATION ERROR

The flight altitude may be determined in a variety of ways. This paper analyzes the method based on the measurement of pressure and temperature. The authors particularly draw attention to the influence of the temperature and pressure measurement errors on the accuracy of the determination of flight altitude. It also depends on the method of approximation. The discussed issues have been described with real data.

Keywords: aviation, flight altitude calculation, atmosphere temperature gradient, standard atmosphere

1. INTRODUCTION

The determination of the flight altitude based on the measurement of temperature and static pressure is a particularly important and significant issue in a practical aspect. The widely applied hypsometric formula [1, 2] assumes that the temperature gradient against the altitude is \( \text{grad}_T = -6.5 \, \frac{\text{K}}{1000 \, \text{m}} \) for the airfield parameters: pressure \( p = 760 \, \text{mmHg} \) and \( t = 15 \, \text{C} \). In reality, the parameters are different. A question arises of how to determine these 3 parameters. The pressure on the airfield elevation can be precisely measured while the measurement of the temperature depending on the location of the sensor on the aircraft (prior to takeoff) may provide varied data (e.g. measurement in the cockpit, on the sunny or shaded side). One of the possibilities of determining the parameters of static
atmosphere, also allowing for the variations of humidity (the gas constant of air changes), is the measurement of temperature and pressure. The measurement data are for the approximation of the temperature, reference pressure and temperature gradient (which is tantamount to the determination of the polytropic exponent n).

2. THE DISTRIBUTION OF PRESSURE IN THE EARTH ATMOSPHERE ACCORDING TO THE POLYTROPIC MODEL

Pressure exerted on the column of liquid of diameter A and height dz is

\[ A \cdot dp = -\rho(z) \cdot g \cdot dz \cdot A, \quad g = \text{const} \]

hence

\[ dp = -\rho(z) \cdot g \cdot dz \]  \hspace{1cm} (1)

where density \( \rho \) fulfills the Clapeyron equation

\[ p = \rho RT \rightarrow \rho = \frac{p}{RT} \]  \hspace{1cm} (2)

thus equation (1) assumes the form

\[ dp = -\frac{g}{RT} \cdot p \cdot dz \]  \hspace{1cm} (3)

or

\[ \frac{dp}{p} = -\frac{g}{RT} \cdot dz \]  \hspace{1cm} (4)

Fig. 1. Changes of the pressure against the height of the column of gas
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Following gas compression, we assume that its temperature changes according to the polytropic equation

\[
\frac{T}{T_0} = \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} \rightarrow T = \frac{T_0}{n-1} \cdot \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}}
\]  

(5)

Substituting temperature \( T \) (4) with (5) we obtain

\[
\frac{p}{p_0}^{\frac{1}{n-1}} \frac{1}{dp} = \frac{g}{R T_0}dz
\]

(6)

or

\[
z = z_0 - \frac{n}{n-1} \cdot \frac{RT_0}{g} \left( \frac{p}{p_0}^{\frac{n-1}{n}} - 1 \right) = z_0 - \frac{n}{n-1} \cdot \frac{R}{g} \cdot (T - T_0)
\]

(7)

The obtained formula is referred to as the hypsometric formula [1]. By differentiating both sides of the equation (7) we obtain

\[
\frac{dT}{g \cdot dT} = -\frac{n}{n-1} \cdot \frac{R}{g} \cdot dT
\]

hence \( \frac{T - T_0}{\text{grad}_T} = z_0 - \frac{T_0}{\text{grad}_T} \cdot \frac{R}{g} \cdot \text{grad}_T \cdot \frac{R}{g}

(7a)

The polytropic exponent for standard atmosphere is found from condition

\[
z = h_{\text{max}} = 11\,000\,\text{m}, \quad \text{then} \quad p(z = h_{\text{max}}) = p_{\text{min}}
\]

(8)
From the equation for standard atmosphere \( p_0 = 760 \text{ mm Hg}, \ g = 9.80665 \text{ m/s}^2, \ R = 287.053 \text{ J/(kgK)}, \ t_0 = 15 ^\circ \text{C}, \ z_0 = 0 \) then the relation between the altitude and pressure is expressed by the formula

\[
z = 44331.5 - 4946.62 \cdot p^{0.190263}
\]

(9)

hence

\[
p = 100 \cdot \left( \frac{44331514 - z}{11880516} \right)^{0.190263}
\]

(10)

and after substituting \( z = h_{\text{max}} \) we obtain the value of pressure \( p_{\text{min}} = p(z = h_{\text{max}}) \). By putting value \( p = p_{\text{min}}, \ z = h_{\text{max}} \) to equation (7) we obtain an equation with an unknown value of the polytropic exponent \( n \), i.e. we have

\[
h_{\text{max}} = z_0 - \frac{n}{n-1} \frac{RT_0}{g} \left[ \left( \frac{p_{\text{min}}}{p_0} \right)^{\frac{n-1}{n}} - 1 \right]
\]

(11)

or after moving the right side to the left side we obtain an equation

\[
f(w) = h_{\text{max}} - z_0 + \frac{1}{w} \frac{RT_0}{g} \left[ \left( \frac{p_{\text{min}}}{p_0} \right)^w - 1 \right] = 0, \ w = \frac{n-1}{n} \]

(12)

Since equation (12) is a non-linear equation due to the unknown \( w \), equation (12) may be solved by the Newton method (in the transversal method version) obtaining the following iterative formula

\[
w_{k+1} = w_k - \frac{f(w_k)}{f'(w_k)}, \ k = 1, 2, \ldots
\]

(13)

with initial quantities \( w_0 = \kappa, \ w_1 = 1.00001 \cdot \kappa \), and the iterative process ends if \( |w_{k+1} - w_k| < \varepsilon \) where \( \varepsilon = 10^{-8} \) was assumed. This condition is fulfilled after 5 iterations and for standard data \( p_0, T_0, R, g, \) polytropic exponent \( n = 1.23493318885 \).

For the polytropic transformation, relation (5) takes place, which upon substituting to (7) provides relation

\[
z = z_0 - \frac{n}{n-1} \frac{RT_0}{g} \left( \frac{T}{T_0} - 1 \right) = z_0 - \frac{n}{n-1} \frac{R}{g} \cdot (T - T_0)
\]

(14)

expressing linear change of temperature with altitude, hence
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$$\frac{dT}{dz} = -\frac{n-1}{n} \cdot \frac{R}{g} = -\frac{1,234933\cdot 9,80665}{1,234933 \cdot 287,053} = -0,64992 \cdot 10^{-2} [\text{K/m}] = -6,4992 [\text{K/km}]$$

(15)

Let us consider the influence of the temperature $T_0$ determination error on the determination of altitude $z$, for error $\delta T_0$ from formula (14)

$$z + \delta z = z_0 - \frac{n}{n-1} \cdot \frac{R}{g} (T - (T_0 + \delta T_0)) = z_0 - \frac{n}{n-1} \cdot \frac{R}{g} (T - T_0) + \frac{n}{n-1} \cdot \frac{R}{g} \cdot \delta T_0$$

(16)

hence, after subtracting relation (4) from (16) we obtain

$$\delta z = \frac{n}{n-1} \cdot \frac{R}{g} \cdot \delta T_0 = \left(0,64992 \cdot 10^{-2}\right)^1 \cdot \delta T_0 = 153,87 \cdot \delta T_0$$

(17)

Thus, determining temperature error $\delta T_0$ has a vital influence on the appropriate determination of altitude $z$.

Let us consider equation (7) and (14)

$$z = z_0 - \frac{n}{n-1} \cdot \frac{RT_0}{g} \left[ \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} - 1 \right]$$

and upon comparison of the sides we have

$$T_0 \left[ \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} - 1 \right] = T - T_0$$

(18)

or

$$u(T_0,T,p) = T_0 \left[ \left( \frac{p}{p_0} \right)^{\frac{n-1}{n}} - 1 \right] - T + T_0$$

(19)

For properly given temperature $T_0$ (fulfilling the polytropic equation), $u(T_0,T,p) = 0$.

At each altitude $z$ the measurement of pressure and temperature with certain accuracy is possible i.e. condition (19) will not be fulfilled. We thus search for temperature $T_0$, so that for subsequent measurements $p_k$ and $T_k$, the sum is mini-

mized
The issue can be generalized for the case when the polytropic exponent \( n \) is unknown too – then functional (21) has the form

\[
J(T_0, w) = \sum_{k=1}^{N_r} \left( u(T_0, T_k, p_k) \right)^2 = \min(T_0)
\]
The first variation $\delta J$ of functional $J$ is not a linear form, namely

$$\frac{1}{2} \frac{\partial J}{\partial T_0} = \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^w \left[ T_0 \left( \frac{P_k}{P_0} \right)^w - T_k \right] = 0$$

(26)

$$\frac{1}{2} \frac{\partial J}{\partial w} = \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^w \cdot \ln \frac{P_k}{P_0} \left[ T_0 \left( \frac{P_k}{P_0} \right)^w - T_k \right] = 0$$

or

$$T_0 \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w} = \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^w \cdot T_k$$

(27)

and following the division from both sides of the equation we obtain a nonlinear equation with unknown $w$

$$F(w) = \frac{\sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w} - \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^w \cdot T_k}{\sum_{k=1}^{N_p} \ln \frac{P_k}{P_0} \left( \frac{P_k}{P_0} \right)^{2w} - \sum_{k=1}^{N_p} \ln \frac{P_k}{P_0} \left( \frac{P_k}{P_0} \right)^w \cdot T_k} = 0$$

(28)

To solve equation (28) we will apply the Newton method, i.e. the subsequent approximations $w_k$ of the root are determined with formula (the derivative is replaced with a differential quotient)

$$w_{k+1} = w_k - \frac{F(w_k)}{F'(w_k)} \approx w_k - \frac{\frac{F(w_k)}{w_k}}{\frac{F(w_k)}{w_k} - \frac{F(w_{k-1})}{w_{k-1}}} = w_k - \frac{w_k}{w_k - w_{k-1}}, \quad k = 1, 2, \ldots$$

(29)

and as the initial value $w_0$ for the isentropic exponent for air we will assume

$$w_0 = \frac{\kappa - 1}{\kappa} = \frac{1,4 - 1}{1,4} = \frac{1}{3,5}$$
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Let the pressure measured $p_k$, $k = 1, \ldots, N_p$ be burdened with error $\delta p_k$ as well as temperature $T_k$ with error $\delta T_k$, then from formula (24) we obtain

$$T_0 + \delta T_0 = \frac{\sum_{k=1}^{N_p} \left( \frac{p_k + \delta p_k}{p_0} \right)^{n-1} \cdot (T_k + \delta T_k)}{\sum_{k=1}^{N_p} \left( \frac{p_k + \delta p_k}{p_0} \right)^{2(n-1)}}$$  \hspace{1cm} (30)

By subtracting relation (24) from relation (30) we obtain

$$\delta T_0 = \frac{\sum_{k=1}^{N_p} \left( \frac{p_k + \delta p_k}{p_0} \right)^{n-1} \cdot (T_k + \delta T_k)}{\sum_{k=1}^{N_p} \left( \frac{p_k + \delta p_k}{p_0} \right)^{2(n-1)}} - \frac{\sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{n-1} \cdot T_k}{\sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{2(n-1)}}$$ \hspace{1cm} (31)

In order to explore the influence of the temperature measurement error $\delta T_k$, $k = 1, \ldots, N_p$ on error $\delta T_0$ let us assume $\delta p_k = 0$, then

$$\delta T_0 = \frac{\sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{n-1} \cdot \delta T_k}{\sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{2(n-1)}}$$ \hspace{1cm} (32)

hence

$$|\delta T_0| \leq \frac{\sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{n-1}}{\sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{2(n-1)}} \cdot \max_k \delta T_k = wz_{-T} \cdot \max_k |\delta T_k|$$ \hspace{1cm} (33)

coefficient $wz_{-T}$
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\[
wz\_T = \sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{n-1} \left( \frac{p_k}{p_0} \right)^{n-1} > 1, \quad \text{because} \quad \left( \frac{p_k}{p_0} \right)^{2(n-1)} > 1 \quad \text{for} \quad n > 1 \quad (34)
\]

has a nature that amplifies the temperature measurement errors. Pursuant to (7)

\[
z = z_0 - \frac{n}{n-1} \cdot \frac{RT_0}{g} \left[ \left( \frac{p}{p_0} \right)^{n-1} - 1 \right] \quad \text{hence} \quad \left( \frac{p}{p_0} \right)^{n} = 1 - (z - z_0) \cdot \frac{n-1}{n} \cdot \frac{g}{RT_0}
\]

and

\[
\left( \frac{p}{p_0} \right)^{2(n-1)} = \left[ 1 - (z - z_0) \cdot \frac{n-1}{n} \cdot \frac{g}{RT_0} \right]^2 \quad p \leq p_{\text{min}} = p(h_{\text{max}})
\]

thus

\[
wz\_T(z_{\text{np}}) = \sum_{k=1}^{N_p} \left[ 1 - (z_k - z_0) \cdot \frac{n-1}{n} \cdot \frac{g}{RT_0} \right]^2 \quad \text{for} \quad n > 1
\]

Figure 2 presents the relation of the temperature error amplification as a function of altitude; it has been assumed that the measurements are performed every 10 m. The nature of function \( wz\_T \) is nearly linear. Since \( \frac{d(wz\_T)}{dz} \approx 0.000011294 \), then the function \( wz\_T \) increment with altitude is miniscule. For the calculations, temperature \( T_0 = 288.15 \) [K] was adopted.

Let us perform the analysis of the influence of the pressure measurement error and the temperature measurement error on the temperature \( T_0 \) determination error. The k-th measurement is burdened with pressure measurement error \( \delta p_k \) and temperature measurement error \( \delta T_k \) which results in error \( \delta T_0 \). Based on formula (24)

\[
T_0 + \delta T_0 = p_0^w \cdot \sum_{k=1}^{N_p} \left( p_k + \delta p_k \right)^w \cdot \left( T_k + \delta T_k \right)
\]

\[
= \sum_{k=1}^{N_p} \left( p_k + \delta p_k \right)^2 \cdot \left( T_k + \delta T_k \right)
\]
Fig. 2. The course of the amplification coefficient (34) for the temperature measurement

hence, by subtracting expression (24) from expression (36) we obtain

$$\delta T_0 = p_0^w \left( \sum_{k=1}^{N_p} (p_k + \delta p_k)^w \cdot (T_k + \delta T_k) - \sum_{k=1}^{N_p} (p_k)^w \cdot T_k \right)$$

(37)

Since $T_0 = T_0(p_1, p_2, ..., p_{N_p}, T_1, T_2, ..., T_{N_p})$ for increments (measurement errors) $\delta p_1, \delta p_2, ..., \delta p_{N_p}, \delta T_1, \delta T_2, ..., \delta T_{N_p}$ the general form of the expression determining increment $\delta T_0$ is as follows (we use the expansion of function $T_0(p_1, ..., p_{N_p}, T_1, ..., T_{N_p})$ in Taylor series in the surroundings of point $(p_1, ..., p_{N_p}, T_1, ..., T_{N_p})$

$$\delta T_0 = T_0(p_1 + \delta p_1, ..., p_{N_p} + \delta p_{N_p}, T_1 + \delta T_1, ..., T_{N_p} + \delta T_{N_p}) - T_0(p_1, ..., p_{N_p}, T_1, ..., T_{N_p}) =$$

$$= \sum_{k=1}^{N_p} \frac{\partial T_0}{\partial p_k} \cdot \delta p_k + \sum_{k=1}^{N_p} \frac{\partial T_0}{\partial T_k} \cdot \delta T_k + ...$$

(38)

Omitting the higher derivatives we have

$$|\delta T_0| \leq \sum_{k=1}^{N_p} \left| \frac{\partial T_0}{\partial p_k} \right| \cdot |\delta p_k| + \sum_{k=1}^{N_p} \left| \frac{\partial T_0}{\partial T_k} \right| \cdot |\delta T_k| \leq \sum_{k=1}^{N_p} \left| \frac{\partial T_0}{\partial p_k} \right| \cdot |\delta p_k| + \sum_{k=1}^{N_p} \left| \frac{\partial T_0}{\partial T_k} \right| \cdot \max |\delta p_k|$$
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\[ + \sum_{k=1}^{N_p} \left| \frac{\partial T_k}{\partial T_k} \right| \cdot \max \left| \frac{\partial T_k}{W} \right| = \delta p \cdot \sum_{k=1}^{N_p} \left| \frac{\partial T_k}{\partial p_k} \right| + \delta T \cdot \sum_{k=1}^{N_p} \left| \frac{\partial T_0}{\partial T_k} \right| = \text{wz}_T (N_p) \cdot \delta T + \text{wz}_p (N_p) \cdot \delta p \]

(39)

where wz_T is the coefficient of amplification of temperature measurement error (formulas (33) and 35)), and wz_p is the coefficient of amplification of the pressure measurement error.

Thus

\[ \text{wz}_p = \sum_{k=1}^{N_p} \left( \frac{\partial T_0}{\partial p_k} \right) \]

(40)

We obtain further derivatives \( \frac{\partial T_0}{\partial p_k} \). Since

\[ T_0 = p_0^w \cdot \sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{n-1} \cdot T_k = p_0^w \cdot \sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{2n-1} \cdot \sum_{k=1}^{N_p} \left( \frac{p_k}{p_0} \right)^{2w-1} \cdot T_k, \quad w = \frac{n-1}{n} \]

then

\[ \frac{\partial T_0}{\partial p_j} = p_0^w \cdot \frac{p_j^{w-1} \cdot T_j \cdot \sum_{k=1}^{N_p} p_k^{2w} - 2 \cdot p_j^{2w-1} \cdot \sum_{k=1}^{N_p} p_k^w \cdot T_k}{\left( \sum_{k=1}^{N_p} p_k^{2w} \right)^2} = \]

\[ = \frac{w}{P_0} \cdot \left( \frac{P_j}{P_0} \right)^{w-1} \cdot T_j \cdot \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w} - 2 \left( \frac{P_j}{P_0} \right)^{2w-1} \cdot \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^w \cdot T_k \]

For the polytropic process

\[ \frac{T}{T_0} = \left( \frac{P}{P_0} \right)^{\frac{n-1}{n}} = \left( \frac{P}{P_0} \right)^w \]

hence
\[ \frac{\partial T_0}{\partial p_j} = w \cdot \frac{T_0}{p_0} \left( \frac{P_1}{P_0} \right)^{2w-1} \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w} - 2 \left( \frac{P_1}{P_0} \right)^{2w-1} \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w} \]

\[ \sum_{k=1}^{N_p} \frac{\partial T_0}{\partial p_j} = w \cdot \frac{T_0}{p_0} \sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w-1} = C_0 \cdot \frac{\sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w}}{\sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w}} \cdot \frac{T_0}{p_0}, \quad C_0 = w \cdot \frac{T_0}{p_0} \] (42)

where constant \( C_0 \) is independent from the measurements.

Eventually, the coefficient of pressure amplification takes the form

\[ wz \_p = C_0 \cdot \psi, \quad \psi = \frac{\sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w-1}}{\sum_{k=1}^{N_p} \left( \frac{P_k}{P_0} \right)^{2w}} > 1 \] (45)

where \( \psi \) is a dimensionless coefficient of pressure amplification.

The collective maximum error of temperature \( T_0 \) determining the error is estimated in advance in the following way

\[ |\delta T| \leq wz \_T \cdot \delta T + wz \_p \cdot \delta p = \]
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\[
\frac{\Delta T}{\Delta p} = \left( \sum_{k=1}^{n} \left( \frac{p_k}{p_0} \right)^{\frac{n-k}{n}} \right) + \frac{n-1}{n} \cdot \frac{T_0}{p_0} \cdot \frac{\Delta p}{p_0} \cdot \frac{n-2}{n} \cdot \frac{\Delta T}{T_0} = \delta T \cdot \phi + \frac{\delta p}{p_0} \cdot \frac{n-1}{n} \cdot T_0 \cdot \psi.
\]

\[\delta p > 0, \quad \delta T > 0\]  \hspace{1cm} (46)

Between the altitude and pressure there is relation (7) from which we have

\[
\frac{p}{p_0} = \left[ 1 - (z-z_0) \frac{n-1}{n} \frac{g}{RT_0} \right]^n.
\]

Estimation (47) was derived assuming that expressions of higher order can be omitted in expansion (38). By putting expression (47) to \(p/p_0\) we obtain a relation

\[
|\Delta T_0| \leq \delta T \cdot \phi (z_1, z_2, \ldots, z_{N_p}) + \frac{\delta p}{p_0} \cdot \frac{n-1}{n} \cdot T_0 \cdot \psi (z_1, z_2, \ldots, z_{N_p})
\]

Figure 3 presents the course of function \(\psi\) (45), \(wz_p = C_0 \cdot \psi\) and \(\phi = wz_T\)

![Coefficient of amplification of temperature and pressure measurement errors](image-url)

Fig. 3. Coefficients of amplification of temperature and pressure measurement errors as a function of altitude.
In the range \( z \in (0, h_{\text{max}}) \), \( h_{\text{max}} = 11,000 \) m from the courses shown in Fig. 2 and 3 we obtain \( \phi \leq 1.134, \psi \leq 2.0883, n = 1.234933 \)

\[
\left| \delta T_0 \right| \leq \delta T \cdot \phi_{\text{max}} + \delta p \cdot \frac{n - 1}{p_0} \cdot \psi_{\text{max}}, \quad \phi = wz \cdot T \tag{48}
\]

while the relative error

\[
\frac{\left| \delta T_0 \right|}{T_0} \leq \frac{\delta T}{T_0} \cdot \phi_{\text{max}} + \frac{\delta p}{p_0} \cdot \frac{n - 1}{n} \cdot \psi_{\text{max}} = 1.134 \frac{\delta T}{T_0} + 0.190239 \frac{2.0883 \delta T}{T_0} + 0.3973 \frac{\delta p}{p_0}
\]

From the above it results that the influence of the relative error of temperature measurement on the relative error of temperature \( T_0 \) determination is \( 1.134/0.3973 \approx 2.854 \) times greater than the relative error of pressure measurement. For \( \delta p = 20 \) Pa, \( \delta T = 0.5 \) K

\[
\frac{\left| \delta T_0 \right|}{T_0} = 1.134 \cdot \frac{0.5}{28815} + 0.3973 \cdot \frac{20}{101300} = 0.001968 + 0.0000784 = 0.002046
\]

hence \( \frac{\left| \delta T_0 \right|}{T_0} = 0.2046\% \) and assuming for example \( T_0 = 299 \) K \( \left| \delta T_0 \right| = 0.589 \) K.

Under actual conditions the parameters of polytropic \( p_0, T_0 \) and exponent \( n \) are different than for standard atmosphere. These parameters can be determined for measurement data of temperature \( T_i \) and pressure \( p_i, i = 1,2,3,\ldots,N \). Namely, the polytropic equation

\[
\frac{p}{p_0} = \left( \frac{T}{T_0} \right)^{n - 1} \tag{49}
\]

after logarithming takes the form

\[
\ln \frac{p}{p_0} - \frac{n}{n - 1} \ln \frac{T}{T_0} = 0 \tag{50}
\]

Since the measurement points in general do not lie on polytropic (49) from relation (50) we have

\[
\delta s_i = \ln \frac{p_i}{p_0} - \frac{n}{n - 1} \ln \frac{T_i}{T_0} \neq 0, \quad i = 1,2,3,\ldots,N \tag{51}
\]

expression (51) is of entropic nature and the minimization of the sum

\[
J = \sum_{i=0}^{N} \left( \delta s_i \right)^2 = \sum_{i=0}^{N} \left( \ln \frac{p_i}{p_0} - \frac{n}{n - 1} \ln \frac{T_i}{T_0} \right)^2 \tag{52}
\]
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allows determination of unknowns $T_0$ and $n$ at a given airfield pressure $p_0$. Assuming

$$u_i = \ln \frac{p_i}{p_0}, \quad x = \frac{n}{n-1}, \quad q_i = \ln \frac{T_i}{T_{\text{ref}}}, \quad y = \ln \frac{T_0}{T_{\text{ref}}}$$

(53)

functional $J$ takes the form (entropic functional)

$$J = \sum_{i=0}^{N} \left( \ln \frac{p_i}{p_0} - \frac{n}{n-1} \cdot \ln \frac{T_i}{T_0} \right)^2 = \sum_{i=0}^{N} (u_i - x(q_i - y))^2 = \sum_{i=0}^{N} (u_i - q_i x + xy)^2$$

(54)

The minimization of functional (54) leads to a non-linear set of equations against $x$ and $y$ variables.

Another method of determination of unknowns $T_0$ and $n$ results from the transformation of expression (50) to the form

$$\frac{n-1}{n} \cdot \ln \frac{p_i}{p_0} - \ln \frac{T_i}{T_0} = 0$$

(55)

hence for measurement points $p_i, T_i, i = 0, 1, \ldots, N$ the error of non-fulfillment of the polytropic equation is

$$\frac{\delta s_i}{\delta s_i} = \frac{n-1}{n} \cdot \ln \frac{p_i}{p_0} - \ln \frac{T_i}{T_0} = \frac{n-1}{n} \cdot \ln \frac{p_i}{p_0} - \ln \frac{T_i}{T_{\text{ref}}} + \ln \frac{T_0}{T_{\text{ref}}}$$

(56)

and the collective error functional (quasi-entropic functional)

$$\bar{J} = \sum_{i=0}^{N} (\delta s_i)^2 = \sum_{i=0}^{N} \left( \frac{n-1}{n} \cdot \ln \frac{p_i}{p_0} - \ln \frac{T_i}{T_{\text{ref}}} + \ln \frac{T_0}{T_{\text{ref}}} \right)^2 = \sum_{i=0}^{N} (z \cdot u_i - q_i + y)^2$$

(57)

Functional (57) is a square functional and his second variation leads to a set of non-linear equations. Figure (4) presents the approximation of measurement data resulting from:

- minimization of functional (57); quasi-entropic approximation,
- minimization of functional (54); entropic approximation.

For these approximations (polytropic curves) based on formula (7) the flight altitude was determined as a function of temperature drop, as shown in Fig. 5.
Fig. 4. Polytropic approximation of measurement data

Fig. 5. Aircraft altitude as a function of temperature drop for reference atmosphere and the approximations of experimental data
The inaccuracy of temperature and pressure measurement significantly influences the temperature gradient which is visible in the inclination of curves $p = p(T)$, Fig. 4 and curves $z = z(T - T_0)$, Fig. 5. For accurate values the curves obtained from the quasi-entropic and entropic approximations are convergent.

3. CONCLUSIONS

The calculations performed in this work indicate a sensitivity of the determination of flight altitude as a function of airfield temperature $T_0$, airfield pressure $p_0$ and the temperature gradient $\text{grad}_T$. For each airfield the $T_0$, $p_0$ and $\text{grad}_T$ parameters are different; they can be determined based on experimental data collected during the flight. In figure 4 the measurement data were approximated according to the quasi-entropic and entropic criterion [4]. For the obtained approximations the flight altitude was determined based on formula (7). In the case of temperature in Fig. 5 courses $z = z(T - T_0)$ were shown for reference atmosphere and 2 approximations. For $T - T_0 = -6.5$ K the differences in the flight altitude are of the order of 100 m against the quasi-entropic approximation. The difference among the approximations is influenced by the accuracy of the temperature measurement, which, for the same pressure (Fig. 4), changes by 3.5 K. Hence, the accuracy of the temperature measurement is of paramount importance.

REFERENCES


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Streszczenie


Słowa kluczowe: lotnictwo, obliczanie wysokości lotu, gradient temperatury atmosfery, atmosfera standardowa